Module 9

Center of Gravity and Centroid

Sections 9.1-9.2 Introduction to the Center of Gravity and Centroid

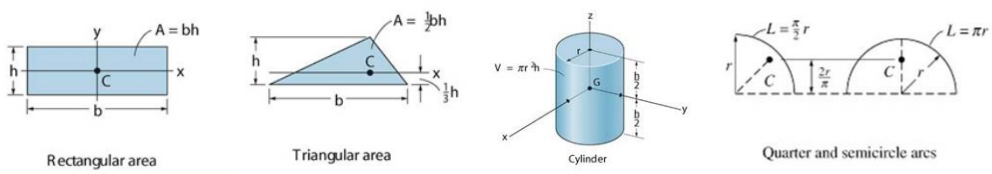
Definitions

- Centroid: geometric center of a body (the point where the distribution of volume/area/length is equal in all directions.
- Center of Gravity: the location of the resultant or total weight of a body
- Center of Mass: the point at which the distribution of mass is equal in all directions

Comparison

- If an object is homogeneous (constant density), then the centroid coincides with the center of mass.
- If an object is in a uniform/constant gravitational field (like an object on Earth), then the center of mass coincides with the center of gravity
- If an object has an axis of symmetry, then the centroid lies on that axis
- In some cases, the centroid of an object may not lie on the object (for example: a donut).

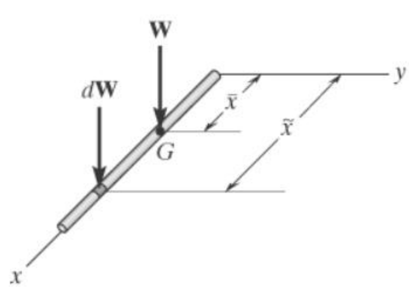
Examples of centroids of common areas



The centroid of these areas can be found using simple geometry! You don't have to memorize these values, since most engineering mechanics books will include tables with the coordinates of centroids for these areas. However, you will notice that the more you practice this topic, the easier it will be to remember and identify these centroids.

In this class, we will use calculus instead of geometry to find the centroid or center of gravity. We will derive a generic approach for the center of gravity of an object.

Finding the center of gravity:



Consider the 1-dimensional rod in the figure, with total weight W acting on the center of gravity $\bar{x}$. If we take an infinitesimally small segment with weight dW at an arbitrary location $\tilde{x}$, then we can determine the total weight mathematically by:

$$W = \int dW$$

Furthermore, the location of the center of gravity can be found by equating the moment of W about the y-axis to the sum of the moments of all the infinitesimally small segments about the same axis:

$$\bar{x}W = \int \tilde{x}dW \Longrightarrow \bar{x} = \frac{\int \tilde{x}dW}{W}$$

If this body were a 2- or 3-dimensional body, we could similarly find the coordinates of the center of gravity in the y- and z-axes as follows:

$$\bar{y} = \frac{\int \tilde{y}dW}{W}$$

$$\bar{z} = \frac{\int \tilde{z}dW}{W}$$

The center of gravity is usually expressed with the variable G, a point with x, y, and z coordinates.

If our gravitational field is constant, we can find the center of mass by substituting $dW = gdm$ and $W = mg$ into our equations:

$$\bar{x} = \frac{\int \tilde{x}gdm}{mg} = \frac{\int \tilde{x}dm}{m}$$

$$\bar{y} = \frac{\int \tilde{y}dm}{m}$$

$$\bar{z} = \frac{\int \tilde{z}dm}{m}$$

These coordinates locate the center of mass, expressed with the variable C_m.

If our body is made of a homogeneous material, then we can find the centroid, C by substituting $dm = \rho dV$ and $m = \rho V$.

$$\bar{x} = \frac{\int \tilde{x}\rho dV}{\rho V} = \frac{\int \tilde{x}dV}{V}$$

$$\bar{y} = \frac{\int \tilde{y}dV}{V}$$

$$\bar{z} = \frac{\int \tilde{z}dV}{V}$$

If our body were a 2-dimensional body, then the centroid would be found in terms of its area:

$$\bar{x} = \frac{\int \tilde{x}dA}{A}$$

$$\bar{y} = \frac{\int \tilde{y}dA}{A}$$

Similarly, if our body were a 1-dimensional body, then the centroid would be found in terms of its length:

$$\bar{x} = \frac{\int \tilde{x} dL}{L}$$

$$\bar{y} = \frac{\int \tilde{y} dL}{L}$$

where $dL = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right) dx = \left(\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\right) dy$.

When dealing with composite bodies (bodies that can be subdivided into simple shapes), we can find the coordinates for the centroid and center of gravity of each "section" and then find the overall centroid and center of gravity by performing a weighted sum. This follows the same process we used when dealing with composite distributed loads.