

Module 1

Properties of Fluids

1.7 Density, Specific Weight, and Specific Gravity

① Density $\rho = \lim_{\delta V \rightarrow \delta V^*} \frac{\delta m}{\delta V} = \frac{m}{V}$ $\rho_{H_2O} = 1000 \text{ kg/m}^3 = 1.94 \text{ slug/ft}^3$

② Specific Weight $\gamma = \frac{F_W}{V} = \frac{mg}{V} = \rho g$ $\gamma_{H_2O} = 9810 \text{ N/m}^3 = 62.4 \text{ lb/ft}^3$

$$F_W = \gamma V$$

③ Specific Gravity $S = \frac{\rho}{\rho_{H_2O}} = \frac{\gamma}{\gamma_{H_2O}}$ $S_{Hg} = 13.56$

④ Specific Volume $v = \frac{1}{\rho}$ Used in Thermodynamics

Approximation of the density of water (as a function of temperature)

$$\rho_{H_2O} = 1000 - \frac{(T - 4)^2}{180}$$

Approximation of the specific weight of water (as a function of temperature)

$$\gamma_{H_2O} = 9800 - \frac{(T - 4)^2}{18}$$

Approximation of the specific gravity of mercury (as a function of temperature)

$$S_{Hg} = 13.6 - 0.0024T$$

Do it yourself: Determine the units for density, specific weight, and temperature for the equations above. Determine the percent error for these equations (by comparing results to tabulated values)

An alternative approximation of the density of water as a function of temperature is

$$\rho_{H_2O} = 1000 - 0.0178(T - 4)^{1.7}$$

In addition to changes due to pressure and temperature, the density of water can vary due to a concentration of a dissolved constituent (e.g. salinity)

1.8 Viscosity

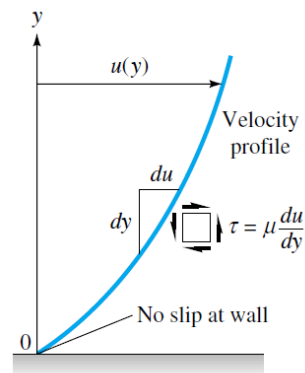
1.8.1 Viscosity of Newtonian Fluids

Definitions

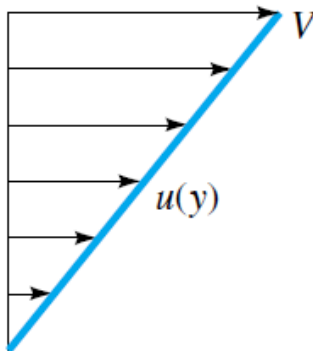
- viscosity (dynamic viscosity, absolute viscosity): a fluid's resistance to deformation from *shear stress*
- no-slip condition: fluid particles in contact with a solid boundary have the same velocity as the boundary
- velocity gradient (strain rate): change of velocity with respect to distance normal to the boundary $\phi = \frac{du}{dy}$

Newton's Law of Viscosity

$$\tau = \mu \frac{du}{dy} = \mu \phi$$



- linear velocity profile: if the thickness of the fluid is very small, we can assume a linear velocity profile



$$\frac{du}{dy} = \frac{u_{max}}{Y}$$

$$u(y) = \frac{u_{max}}{Y} y$$

1.8.2 Viscosity of Non-Newtonian Fluids

Definitions:

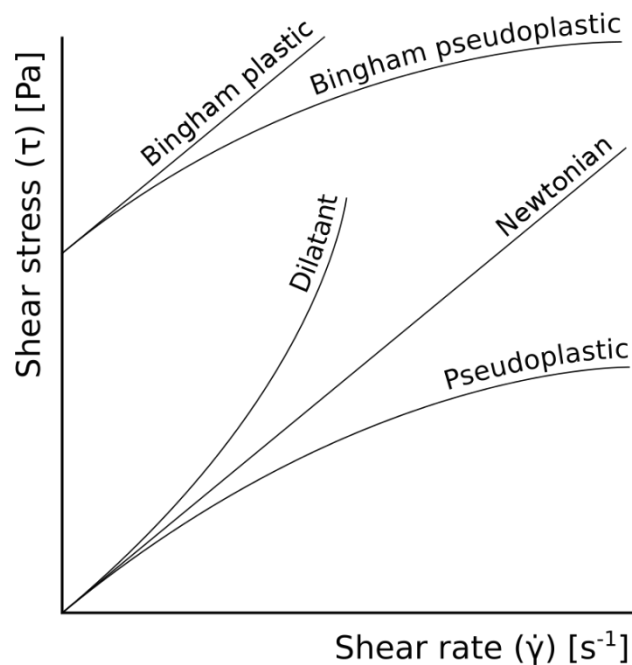
- Newtonian fluid: shear stress is directly proportional to strain rate; $\tau \propto \frac{du}{dy}$ (e.g. air, water, oil)
- Non-Newtonian fluid: shear stress is not proportional to strain rate (e.g. blood, whipped cream)
- Dilatant (shear thickening): viscosity increases with shear stress (e.g. quicksand, oobleck)
- Pseudoplastic (shear thinning): viscosity decreases with shear stress (e.g. ketchup, paint)
- Ideal plastic (Bingham plastic): requires a minimum shear stress to cause motion
- Bingham pseudoplastic: requires a minimum shear stress and behaves like a pseudoplastic (e.g. toothpaste)
- Kinematic viscosity (momentum diffusivity): $\nu = \mu/\rho$ (see Appendix A, p. 842-843, Hibbeler)

The relationship between shear stress and strain rate in non-Newtonian fluids takes one of the following forms:

$$\tau = K \left(\frac{du}{dy} \right)^n = \left[K \left(\frac{du}{dy} \right)^{n-1} \right] \frac{du}{dy} = \mu_{ap} \frac{du}{dy}$$

...where K is the consistency index, and n is the power-law index (or flow behavior index). This relationship can be expressed in terms of Newton's law of viscosity by reducing the variable viscosity to a value called apparent

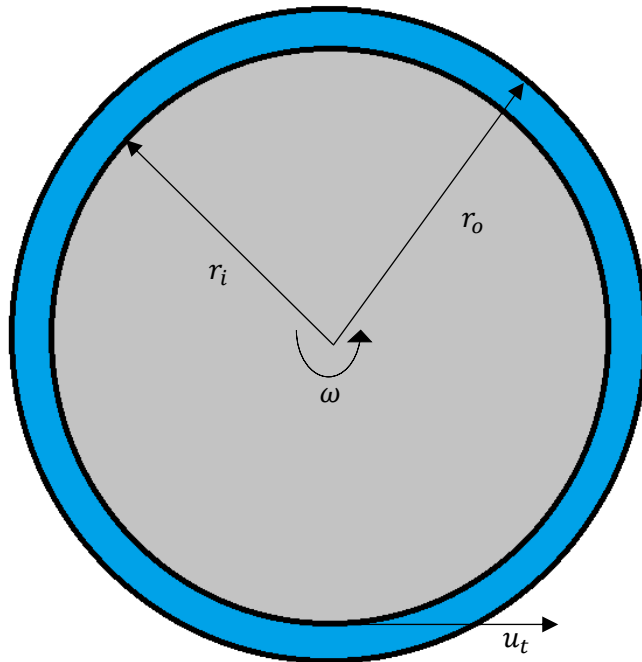
viscosity, $\mu_{ap} = K \left(\frac{du}{dy} \right)^{n-1}$.



1.8.3 Measuring Viscosity with Viscometers

Viscometer: an instrument used to measure viscosity.

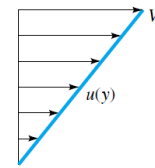
Consider two concentric cylinders with a fluid between them. The thickness of the fluid is Y , and the inner cylinder rotates at an angular velocity ω :



$$Y = r_o - r_i$$

$$u_t = \omega r_i$$

If Y is very small, we can assume that the velocity gradient is linear:



$$u = \frac{\omega r_i}{Y} y$$

$$\frac{du}{dy} = \frac{\omega r_i}{Y}$$

In order to rotate the inner cylinder, a torque T is needed. Recall that:

$$T = F r_i$$

$$F = \tau A$$

$$A = 2\pi r_i h$$

$$\tau = \mu \frac{du}{dy}$$

Therefore:

$$T = 2\pi r_i^2 h \mu \frac{du}{dy}$$

$$T = \frac{2\pi r_i^3 h \mu \omega}{Y}$$

1.8.4 Variation of Viscosity with Temperature

- Viscosity varies with temperature, not pressure.
- Viscosity of gases increases with temperature.
- Viscosity of liquids decreases with temperature

The viscosity of liquids can be approximated by Andrade's Equation

$$\mu = C e^{B/T}$$

where C and B are empirical constants and T is the absolute temperature.

The viscosity of gases can be approximated by the Sutherland equation

$$\mu = \frac{dT^{3/2}}{T + e}$$

where d and e are empirical constants. For air under standard engineering conditions, $d = 1.458 \times 10^{-6} \text{ kg}/(\text{m} \cdot \text{s} \cdot \text{K}^{0.5})$ and $e = 110.4 \text{ K}$. If the equation is applied at a reference condition, it becomes

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2} \frac{T_0 + e}{T + e}$$

As an alternative, a power law relationship is sometimes assumed:

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^n$$

where the exponent n is commonly taken as 0.7.

1.9 Compressibility

1.9.1 Bulk Modulus

Definitions

- compressibility: a fluid's resistance to deformation from normal stress.
- Bulk Modulus of Elasticity: the reciprocal of compressibility

Volumetric Bulk Modulus of Elasticity

$$E_V = -V \frac{\partial P}{\partial V} \cong -\frac{\Delta P}{\Delta V/V}$$

Unit-mass Bulk Modulus of Elasticity

$$E_V = \rho \frac{\partial P}{\partial \rho} \cong \frac{\Delta P}{\Delta \rho/\rho}$$

We can use the bulk modulus to determine the speed of sound! (we will learn more about this in a future lesson)

$$c = \sqrt{\frac{\partial P}{\partial \rho}} = \sqrt{\frac{E_V}{\rho}}$$

Some fluids exhibit very little change with respect to pressure, so we treat them as if they had no compressibility. If temperature doesn't change either, then we can take the fluid's density to be constant with respect to pressure; we call such fluids incompressible fluids.

1.9.2 Empirical Compressibility of Liquids

If we neglect changes in temperature, the density of a liquid can be related to pressure empirically by:

$$\frac{P}{P_a} \approx (B + 1) \left(\frac{\rho}{\rho_a} \right)^n - B$$

...where ρ_a is the density of the fluid under standard engineering conditions, P_a is the atmospheric pressure, and B and n are empirical values.

For water, $B \approx 3000$ and $n \approx 7$.

1.10 Surface Tension

1.10.1 Surface Tension Theory

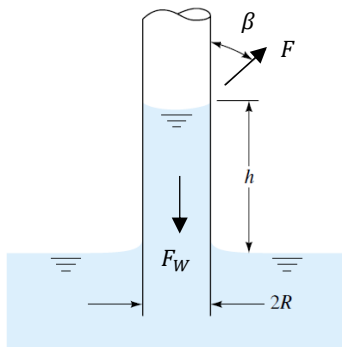
Definitions

- surface tension: a measure of force per unit length in a liquid surface
- surface: the portion of a liquid in contact with a gas
- capillary rise/drop: liquid column that results from forces of attraction between a fluid and a solid
- static rise/drop: liquid column that results from pressure forces
- contact angle: the angle of the direction in which a force of attraction acts. It depends on the liquid, solid, and gas in contact.

$$\sigma = \frac{F}{L} \Rightarrow F = \sigma L$$

1.10.2 Capillary Rise and Drop

Consider a tube inserted in water:



The forces of attraction F between the water and the tube will cause it to “rise”.

However, the weight of the water in the tube will exert a force in the downward direction.

The force of attraction is a function of surface tension and contact angle β .

$$F = \sigma L \cos \beta$$

The system will reach equilibrium when $F = F_W$

$$\sigma L \cos \beta = \gamma V$$

$$\sigma \pi D \cos \beta = \gamma \frac{\pi}{4} D^2 h$$

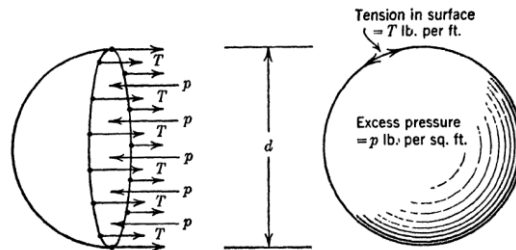
Note that depending on the fluids and solids in contact, the cosine of the contact angle may be negative, which means we have a “drop” instead of a rise. This happens with mercury.

1.10.3 Liquid Droplets

Consider a small liquid droplet with diameter D . In order for this droplet to maintain its shape, a difference in pressure must exist between the external pressure and the internal pressure. When the external pressure is atmospheric, then this difference is called gage pressure.

$$P_i - P_o \neq 0$$

Analyzing the forces acting on half of the droplet results in a force from the surface tension, acting along the circumference, and a force from gage pressure acting on the circular area.



$$\sum F = F_\sigma - F_P = 0$$

$$\pi D \sigma = \frac{\pi}{4} D^2 P$$

$$P = \frac{4\sigma}{D}$$

This shows that the gage pressure inside a liquid droplet is inversely proportional to the diameter of the droplet.

We can also apply this analysis to droplets with double curvature. Consider a small element of dimensions dx dy on a surface of a droplet with curvature radii R_1 and R_2 . The sum of forces normal to the element becomes:

$$\sum F = F_\sigma - F_P = 0$$

$$P \, dx \, dy = 2\sigma dy \sin \alpha + 2\sigma dx \sin \beta$$

...where α and β are small angles corresponding to the radii and half of the element dimensions: so that

$$\sin \alpha = \frac{\frac{dx}{2}}{R_1} = \frac{dx}{2R_1}$$

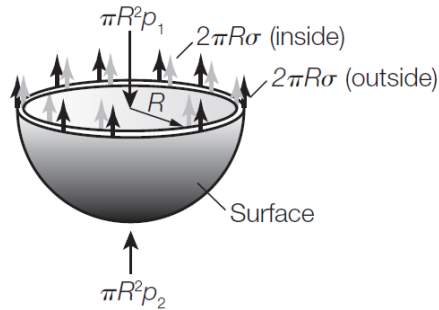
$$\sin \beta = \frac{\frac{dy}{2}}{R_2} = \frac{dy}{2R_2}$$

Substituting these values in the equation above yields:

$$P = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

1.10.4 Bubbles in Gases

For bubbles in gases (i.e. air), there is a thin layer of liquid between the inside and outside of the bubble, which creates two surfaces in which surface tension acts. Keeping in mind that $P_1 - P_2 = P$ Analyzing the forces acting on half of a bubble of diameter D yields:



$$\sum F = F_\sigma - F_p = 0$$

$$\pi D\sigma + \pi D\sigma + \frac{\pi}{4}D^2P_2 - \frac{\pi}{4}D^2P_1 = 0$$

$$2\pi D\sigma = \frac{\pi}{4}D^2P$$

$$P = \frac{8\sigma}{D}$$

From this equation, we see that the pressure inside a bubble is twice as large as the pressure inside a droplet of the same diameter.

1.10.5 Bubbles in Liquids

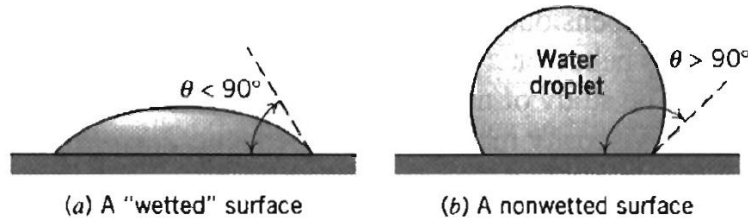
The difference in pressure for bubbles in liquids yields the same value as that of a liquid droplet. However, the external pressure will be higher than the atmospheric pressure (as a hydrostatic pressure is also added to it), and so the equation takes the following notation

$$\Delta P = P_i - P_o = \frac{4\sigma}{D}$$

A bubble in a liquid must have an inside pressure higher than that on the outside. Therefore, in order to generate a bubble, a pressure of $P_i = P_o + \Delta P = P_o + \frac{4\sigma}{D}$ is necessary where P_o is the pressure in the liquid at the location the bubble is to be created.

1.10.6 Droplets on Solid Surfaces

When a liquid droplet contacts a solid surface, the angle formed between the liquid and solid surface is called the contact angle. When $\theta > 90^\circ$, the liquid is a wetting liquid, otherwise, it is a nonwetting liquid.



1.11 Vapor Pressure

1.11.1 Vapor Pressure Theory

- vapor pressure: the pressure of the gaseous phase of the fluid that is in contact with the liquid phase of the fluid and in equilibrium, P_V
- Vapor pressure increases as temperature increases.
- When the absolute pressure in a liquid is less than its vapor pressure ($P_{abs} < P_V$), it evaporates.
- When the absolute pressure in a gas is greater than its vapor pressure ($P_{abs} > P_V$), it condenses.
- The vapor pressure of water at 100°C is 101.3 kPa , which means that, at sea level, water will evaporate at 100°C .

1.11.2 Cavitation

Definitions:

- Cavitation: formation of low-pressure pockets (vapor cavities) when the pressure within a liquid is equal or lower than the vapor pressure.
- Boiling: the spontaneous formation of vapor cavities within a liquid
- Boiling occurs when the temperature of a liquid is raised or when the pressure of a liquid is lowered.

Note: Vapor pressure increases with temperature, so at higher fluid temperatures there is greater danger of cavitation.

Cavitation can cause damage to conduits and propellers, but it can also have positive uses, such as in ultrasonic cleaning and supercavitating torpedoes, and shock wave lithotripsy.

The potential for cavitation in a flowing liquid is measured by the cavitation number:

$$Ca = \frac{P - P_V}{\frac{1}{2}\rho v^2}$$

1.11.3 Humidity

- saturation pressure: the pressure at which a pure substance changes phase, P_{sat}
- In pure substances, the vapor pressure and saturation pressure are the same.

In mixtures, the partial pressure of each component must be equal to their respective vapor pressures in order to achieve phase equilibrium. When this occurs, the system is said to be saturated. Thus, the rate of evaporation from open water bodies (e.g. lakes) is controlled by the difference between the vapor pressure and the partial pressure.

In hydrologic and agricultural applications, the relative humidity is used to relate the vapor pressure and partial pressures of a substance:

$$RH = \frac{P}{P_V} \times 100$$

Based on this equation, evaporation can only occur when the relative humidity is less than 100%. Some sources use the variables e and e_s to denote partial pressure and vapor pressure, respectively.

1.12 Laws of Conservation

If we define a system as a fixed quantity of matter, then the following laws of conservation apply:

Conservation of Mass: Matter cannot be created nor destroyed.

By definition the mass in a system is fixed and cannot change, so:

$$\left(\frac{dm}{dt}\right)_{sys} = 0$$

Conservation of Momentum: The momentum in an isolated system remains constant.

Newton's second law of motion states that the change of motion of a body is proportional to the net force acting on it, and acts along the direction of that force. This can be expressed in vector form as $\sum \vec{F} = d(m\vec{v})/dt$. If a system is isolated, there will be no external forces acting on it, so the law of conservation of momentum can be expressed in vector form as $(d(m\vec{v})/dt)_{sys} = 0$. However, in this course, most systems we will study are not isolated, so our study of conservation of momentum will lead us to use the more general form of Newton's second law.

Furthermore, because a system is a fixed quantity of matter, we can treat mass as a constant and express the law as:

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = m \frac{d\vec{v}}{dt} = m\vec{a}$$

Conservation of Energy: The total energy in an isolated system remains constant.

$$E_2 - E_1 = Q_{1-2} - W_{1-2}$$

1.13 Ideal Gas Law

An ideal gas (or perfect gas) has the following relationship between pressure, temperature, and density:

$$P = \rho RT$$

$$PV = mRT$$

where pressure and temperature are in their absolute scales, and R is the gas constant. The gas constant is determined by

$$R = \frac{R_u}{M}$$

where R_u is the universal gas constant, $8.314 \text{ kJ}/(\text{kmol} \cdot \text{K}) = 49,710 \text{ ft} \cdot \text{lb}/(\text{slugmol} \cdot ^\circ\text{R})$, and M is the molar mass. In prior decades, R_u was expressed in terms of pounds instead of slugs, so the ideal gas law would be written in terms of specific weight instead of density. We will not worry about this in class but students should be aware of this when reading older literature on the subject.

When a mixture contains various gases (such as in air), Dalton's Law of Partial Pressures states that each gas exerts its own pressure as if the others were not present.

In engineering applications, most gases can be treated as ideal gases. However, vapors cannot be treated as ideal gases.

1.14 First Law of Thermodynamics

- First Law of Thermodynamics: when a system changes from one state to another, its energy content changes by energy exchange with its surroundings.
- Energy exchange: occurs in the form of heat transfer or work

In this course, we will define heat transfer to the system as positive and work done by the system as negative, therefore:

$$Q_{1-2} - W_{1-2} = E_2 - E_1$$

where Q_{1-2} is the amount of heat transfer to the system, W_{1-2} is the work done by the system, and E_2 & E_1 are the total energies in the second and first state, respectively. The total energy consists of internal, kinetic, and potential energy:

$$E = m\tilde{u} + \frac{mv^2}{2} + mgz = m \left(\tilde{u} + \frac{v^2}{2} + gz \right)$$

where $m\tilde{u}$ is the internal energy (and by default, \tilde{u} is the internal energy per unit mass), $\frac{mv^2}{2}$ is the kinetic energy (where v is the velocity), and mgz is the potential energy (weight times vertical distance).

- Extensive property: a thermodynamic property that depends on a system's mass. (e.g. momentum, energy)
- Intensive property: a thermodynamic property that is independent of a system's mass. It can be obtained by dividing the extensive property of a system by its mass. (e.g. temperature, pressure)

Notice that energy is an extensive property, but its intensive property, known as specific energy, can be obtained by dividing the energy of a system by its mass.

$$e = \tilde{u} + \frac{v^2}{2} + gz$$

- Isolated system: a system that is thermodynamically disconnected from its surroundings.

In an isolated system $Q_{1-2} = 0$ and $W_{1-2} = 0$, therefore:

$$E_1 = E_2$$

This equation represents the conservation of energy.

- Work: a measure of energy transfer that occurs when an object is moved over a distance by an external force.

$$W_{1-2} = \int_{l_1}^{l_2} F \, dl = \int_{l_1}^{l_2} PA \, dl = \int_{V_1}^{V_2} P \, dV$$

1.15 Other Thermodynamic Properties

1.15.1 Enthalpy

- Enthalpy: the sum of a system's internal energy and the product of its pressure and volume

$$H = m\tilde{u} + PV$$

Its corresponding intensive property is:

$$h = \frac{H}{m} = \tilde{u} + \frac{P}{\rho}$$

1.15.2 Specific Heat

- Constant-pressure specific heat: a property used to calculate the change in enthalpy in an ideal gas

$$\Delta h = \int c_p \, dT$$

$$dh = c_p \, dT$$

- Constant-volume specific heat: a property used to calculate the change in internal energy in an ideal gas

$$\Delta \tilde{u} = \int c_v \, dT$$

$$d\tilde{u} = c_v \, dT$$

The relationship between the two specific heats can be found through applying the ideal gas law to the enthalpy equation and differentiating:

$$h = \tilde{u} + \frac{P}{\rho} = \tilde{u} + RT$$

$$dh = d\tilde{u} + RdT$$

$$c_p dT = c_v dT + RdT$$

$$c_p = c_v + R$$

The ratio of specific heats (or adiabatic exponent) can be expressed as:

$$k = \frac{c_P}{c_V}$$

In many situations, the specific heats can be assumed to be constant. For air and other diatomic gases, $k = 1.4$.

1.15.3 Isothermal and Isentropic Processes

- Quasi-equilibrium process (quasi-static process): a process in which pressure, temperature, and other properties are constant at any instant throughout the system (e.g. compression and expansion in the cylinder of an internal combustion engine)
- Isothermal process: a process in which temperature is constant
- Isobaric process: a process in which pressure is constant
- Adiabatic Process: a quasi-equilibrium process in which there is no heat transfer.
- Isentropic Process: a frictionless and reversible adiabatic process

For perfect gases, the relationship between pressure and density can be given by:

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^n$$

which also gives the following relationships:

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(n-1)/n} = \left(\frac{\rho_1}{\rho_2}\right)^{n-1}$$

where n is a nonnegative value from zero to infinity that is determined by the process to which the gas is subjected.

An isothermal process occurs when the temperature does not change from one state to another, so $T_1 = T_2$ and $\frac{T_1}{T_2} =$

1. From the equation above, this also means that $\left(\frac{\rho_1}{\rho_2}\right)^{n-1} = 1$ so that $n = 1$, and:

$$\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1}$$

This equation can also be expressed as Boyle's Law, which states that, for an isothermal process:

$$\frac{P}{\gamma} = \text{const}$$

An isentropic process occurs when entropy is constant, and n is the ratio of specific heats ($n = k$):

$$\frac{P_1}{P_2} = \left(\frac{\rho_1}{\rho_2}\right)^k$$

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(k-1)/k} = \left(\frac{\rho_1}{\rho_2}\right)^{k-1}$$

An isobaric process occurs when $\frac{P_1}{P_2} = 1$ and $n = 0$. An expansion with friction occurs when $n < k$. A compression with friction occurs when $n > k$.

1.15.4 Compressibility of Perfect Gases

In thermodynamic applications, the volumetric bulk modulus can be expressed as a function of specific volume:

$$E_V = -V \frac{\partial P}{\partial V} \left(\frac{m}{m} \right) = - \left(\frac{v}{\partial v} \right) \partial P$$

The pressure/density relationship for perfect gases can also be expressed as a function of specific volume:

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1} \right)^n = \left(\frac{v_1}{v_2} \right)^n$$

$$P_1 v_1^n = P_2 v_2^n = \text{const} \therefore P v^n = \text{const}$$

Differentiating this equation gives:

$$n P v^{n-1} dv + v^n dP = 0$$

$$dP = - \frac{n P v^{n-1}}{v^n} dv = - \frac{n P}{v} dv$$

Combining with the volumetric bulk modulus gives:

$$E_V = - \left(\frac{v}{dv} \right) \left(- \frac{n P}{v} dv \right) = n P$$

So, for an isothermal process: $E_V = P$ and for an isentropic process, $E_V = k P$.

The inverse of the bulk modulus is called the isothermal compressibility and is denoted by:

$$\alpha = \frac{1}{E_V} = - \frac{1}{v} \frac{\partial v}{\partial P}$$

1.15.5 Speed of Sound

If a fluid is inelastic ($E_V \rightarrow \infty$), pressure disturbances are transmitted instantaneously. In elastic fluids, small pressure changes travel at a finite velocity, known as celerity. The celerity is often called the sonic velocity, acoustic velocity, or speed of sound.

- Sound: a pressure wave that travels through a medium that can be perceived audibly

For small pressure waves travelling in a perfect gas at low frequencies (e.g. a sound wave), the disturbance is so small and rapid that heat exchange may be neglected, and the wave speed is given by an isentropic process so that:

$$c = \sqrt{\frac{\partial P}{\partial \rho}} = \sqrt{\frac{k P}{\rho}} = \sqrt{k R T}$$

where k is the ratio of specific heats. For high frequencies, entropy is not constant, and the equation becomes:

$$c = \sqrt{\frac{\partial P}{\partial \rho}} = \sqrt{\frac{P}{\rho}} = \sqrt{R T}$$

The ratio of an object's speed to the speed of sound is called the Mach number, Ma :

$$\text{Ma} = \frac{v}{c}$$

If $\text{Ma} < 1$, then an object is at subsonic speed, and if $\text{Ma} > 1$, an object is at supersonic speed. For an object at constant speed, the Mach number decreases as the temperature increases. Fluids with $\text{Ma} \leq 0.3$ can be assumed to be incompressible.

1.15.6 Heat Transfer in an Isothermal Expansion

The ideal gas law can be expressed in terms of the number of moles of the gas, and the universal gas constant as follows:

$$PV = mRT = m \frac{R_u}{M} T = nR_u T$$

where n is the number of moles of the gas and is determined by $n = m/M$.

For an isothermal expansion of an ideal gas, the heat transfer can be determined by combining the 1st Law of Thermodynamics with the Ideal Gas Law (expressed in terms of number of moles):

$$Q_{1-2} - W_{1-2} = E_2 - E_1$$

Where $W_{1-2} = \int_{V_1}^{V_2} P dV$ and $E_2 - E_1 = 0$

$$Q_{1-2} = W_{1-2} = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} \frac{nR_u T}{V} dV = nR_u T \int_{V_1}^{V_2} \frac{1}{V} dV = nR_u T \ln \frac{V_2}{V_1}$$

1.15.7 Coefficient of Thermal Expansion

The compressibility of a fluid considers changes in pressure, and coefficients such as the bulk modulus consider constant fluid properties throughout (i.e. constant temperature). The coefficient that takes into account changes in temperature (while keeping pressure constant) is the coefficient of thermal expansion and is denoted as:

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right) \cong \frac{\Delta v/v}{\Delta T}$$

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right) \cong -\frac{\Delta \rho/\rho}{\Delta T}$$

The thermal expansion coefficient for an ideal gas is equivalent to the inverse of the temperature:

$$\beta = \frac{1}{T}$$

In natural convection currents, the temperature and density of the fluid body that surrounds a finite hot or cold region is given a subscript of ∞ , and the thermal expansion coefficient is approximated by:

$$\beta \approx -\frac{(\rho_\infty - \rho)/\rho}{T_\infty - T}$$

The combined effects of pressure and temperature changes on the volume change of a fluid is determined by taking the specific volume to be a function of pressure and temperature, which can then be related to the isothermal compressibility and the thermal expansion coefficient:

$$dv = \frac{\partial v}{\partial T} dT + \frac{\partial v}{\partial P} dP = \beta v dT - \alpha v dP = v(\beta dT - \alpha dP)$$

The fractional change in volume can be approximated as:

$$\frac{dv}{v} = \beta dT - \alpha dP$$

$$\frac{\Delta v}{v} = \beta \Delta T - \alpha \Delta P$$

Similarly, for density:

$$\frac{d\rho}{\rho} = \alpha dP - \beta dT$$

$$\frac{\Delta \rho}{\rho} = \alpha \Delta P - \beta \Delta T$$

1.15.8 Latent Heat

- Latent heat: energy per unit mass that is absorbed or released by a fluid upon a change in phase at a constant temperature and pressure.
- Latent heat of vaporization (enthalpy of vaporization), L_v : amount of heat required to convert a unit mass of a fluid from the liquid to a vapor phase at a given temperature.
- Latent heat of fusion (enthalpy of fusion), L_f : amount of heat required to convert a unit mass of a solid to a liquid at the melting point.

1.16 Standard Atmosphere

Atmospheric properties vary with time and latitude, therefore, engineering calculations are based on a standard atmosphere, which is at 40° latitude. The standard atmosphere is a set of standard values (pressure, temperature, etc.) that were adopted in the 1920's to standardize aircraft instruments and aircraft performance. These values have been extended and improved, with the latest accepted iterations being the International Civil Aviation Organization (ICAO) standard atmosphere adopted in 1964, the International Standards Organization (ISO) standard atmosphere adopted in 1973, and the United States (US) standard atmosphere adopted in 1976.

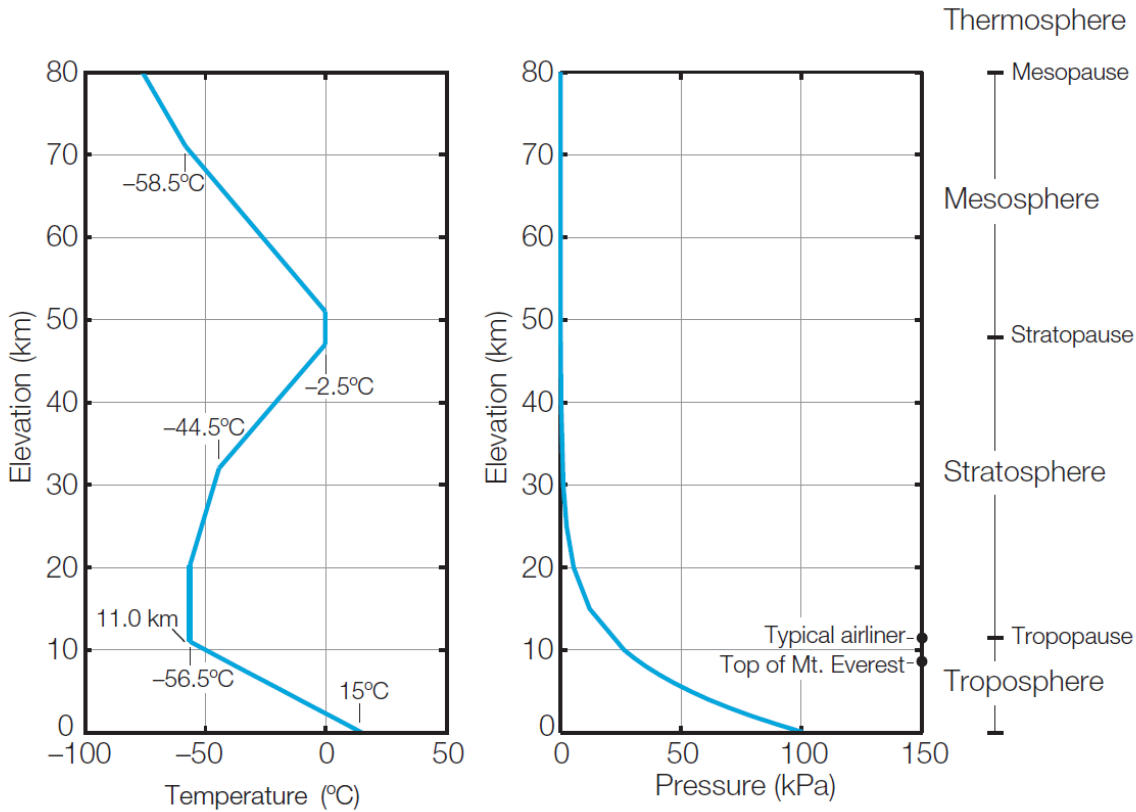
The atmosphere is divided into four layers: the troposphere, stratosphere, mesosphere, and the ionosphere (which is itself composed of the thermosphere, exosphere, and part of the mesosphere). In the troposphere, at altitudes of 0 – 11.02 km (36,200 ft), temperature decreases linearly at a lapse rate α . The temperature can be expressed as a function of elevation with the equation

$$T(z) = T_0 - \alpha z$$

where $T_0 = 288 \text{ K} = 518 \text{ }^\circ\text{R}$ and $\alpha = -6.489 \text{ }^\circ\text{C}/\text{km} = -3.560 \text{ }^\circ\text{F}/(1000 \text{ ft})$. In the stratosphere, at altitudes of 11.02 – 20.06 km, the temperature remains constant at $-56.5 \text{ }^\circ\text{C} (-69.7 \text{ }^\circ\text{F})$. In the mesosphere, at altitudes of 20.06

– 31.16 km, the temperature increases linearly to $-44.5\text{ }^{\circ}\text{C}$, then from 31.16 – 47.35 km, it increases linearly to $-2.5\text{ }^{\circ}\text{C}$ ($27.5\text{ }^{\circ}\text{F}$). From 47.35 – 51.41 km, the temperature remains constant. From 51.41 – 71.80 km, the temperature decreases linearly to $-58.5\text{ }^{\circ}\text{C}$. From 71.80 – 86.00 km, the temperature decreases linearly to $-86.28\text{ }^{\circ}\text{C}$ ($-123.30\text{ }^{\circ}\text{F}$).

The standard pressure decreases rapidly to almost zero at an altitude of 30 km (98,000 ft). The pressure profile can be determined using fluid statics calculations, which are covered in future lessons.



1.17 Other Fluid Properties

1.17.1 Salinity

When salt is added to water, its density is increased and its freezing point is decreased. This is why salt is added to roads to prevent ice formation. The salt content in a body of water is measured by its salinity, which is the ratio of the weight of the salt to the total weight of the mixture:

$$S = \frac{F_{W\text{ salt}}}{F_{W\text{ mixture}}}$$

The average salinity of seawater is 0.035, commonly expressed as 35‰ (parts per thousand).

